GRAPH THEORY

Introduction:

DEFINITION (GRAPH):

A graph G is mathematical structure consisting of an ordered pair (V, E), where V is a non empty finite set whose elements are called points (or) vertices (or) nodes and $E(G)$ is a set of unordered pairs of distinct elements of $V(G)$. The elements of $E(G)$ are called the edges (or) lines of the graph.

Several examples of graph and their corresponding picture follows:

1. 

Fig: 1.1

The vertex set V = {u,v}.
The edge set E = {e}

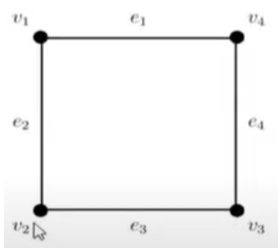
2. 

Fig : 1.2

The vertex set $V = \{v_1, v_2, v_3, v_4\}$.
The edge set $E = \{e_1, e_2, e_3, e_4\}$.

BASIC TERMINOLOGIES RELATED TO GRAPH:

1. SELF- LOOP IN A GRAPH :
We observe that the definition of a graph allows an edge to be of the form (v_i, v_i). Such an edge having the same vertex as both its end vertices is called a self-loop. For example, Edge e_5 in the graph of Fig 1.3 is a self – loop.

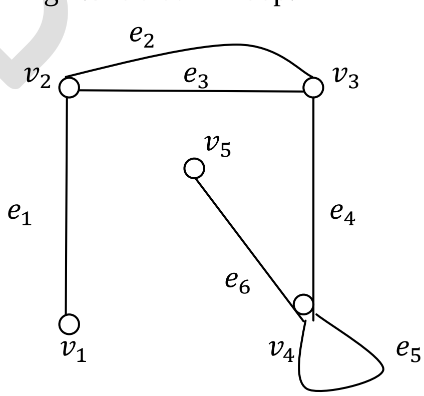


Fig: 1.3 Graph with five vertices and six edges.

2. **PROPER EDGE** : An edge which is not self loop is called proper edge. For example, in Fig 1.3, the edges e_1, e_2, e_3, e_4 and e_5 are proper edges.
3. **PARALLEL EDGES**: A collection of two or more edges having identical end points are called as multi edge. For example, Edges e_3 and e_4 in Fig 1.4 are parallel edges.

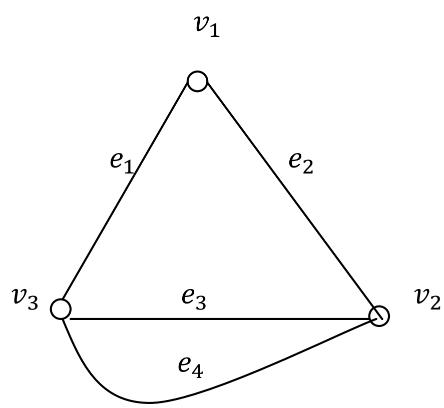


Fig 1.4: Graph with three vertices and four edges

4. **INCIDENT EDGE** : Let e_k be an edge joining two vertices v_i and v_j of a graph G. Then the edge e_k is said to be incident on each of its end vertices v_i and v_j . For example, in the graph of Fig 1.4, edge e_2 is incident on vertices v_1 and v_2 .
5. **ADJACENT VERTICES**: Two vertices in a graph are said to be adjacent if there exists an edge joining the vertices.

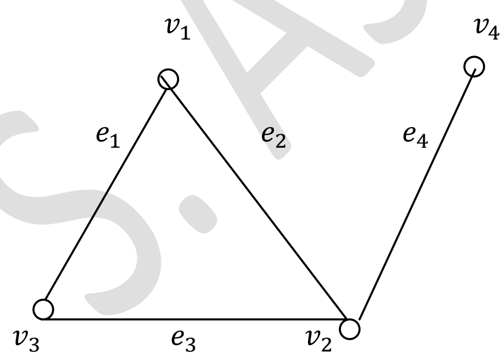


Fig: 1.5 Graph with four vertices and four edges

Different types of Graphs:

1. **TRIVIAL GRAPH** : A graph containing only one vertex and no edge.

   Example: 

2. **NULL GRAPH** : A graph containing n vertices and no edges.

   Example: 

3. **SIMPLE GRAPH** : A graph that does not contain any self- loop and multiedge.

Example :

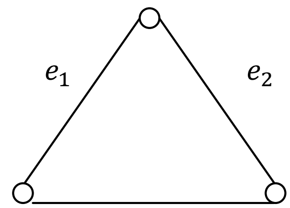
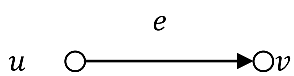


Fig 1.6 Graph with three vertices and three edges

4. **MULTI GRAPH**: A graph that does not contain any self loop but contains parallel edge is called multi graph. Fig 1.4 is an example of multi graph.

5. **DIRECTED GRAPH**: A graph which contains the direction of edges is called directed graph. A directed graph is also called as Digraph. For example,

i)



ii)

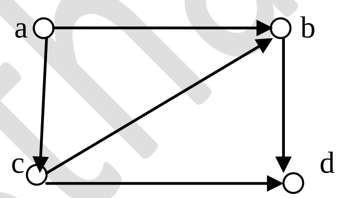


Fig 1.7, Graph with two vertices and one edge

Fig 1.8, Graph with four vertices and five edges.

A graph which is not directed is called as undirected graph.

6. **PSEUDO GRAPH**: A graph that contains both self –loop and multi edge is called pseudo graph.

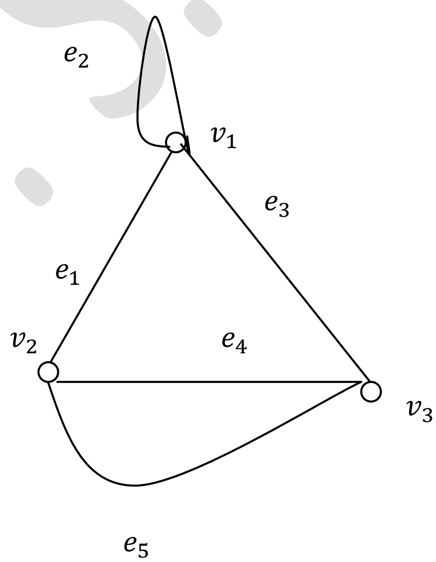


Fig 1.9, Graph with three vertices and five edges.

BASIC GRAPH PARAMETERS AND DEGREE:

Let $G = (V, E)$ be a graph. We define the following parameters of G.

• The graph G is non-trivial if it contains at least one edge, i.e., $E \neq \emptyset$. Equivalently, G is non-trivial if G is not an empty graph.

• The order of G, denoted by $|G|$, is the number of vertices of G, i.e., $|G| = |V|$.

• The size of G, denoted by $||G||$, is the number of edges of G, i.e., $||G|| = |E|$.

Note that if the order of G is n, then the size of G is between 0 and $C(n, 2)$.

- The degree of a vertex v of G, denoted by d(v) or deg (v) , is the number of edges incident to v.
  - A vertex of degree 1 is called pendant vertex.
  - A vertex of degree 0 is called isolated vertex.

APPLICATION OF GRAPH THEORY:

The objective of study of graph lies in its applicability to some problem in almost every conceivable discipline using graph models. It has made its uses felt not only in mathematical field only but also in diverse fields like economics, biology, psychology, computers, bridge problem and puzzles. Graphs can be used to study the structures of the world wide web.